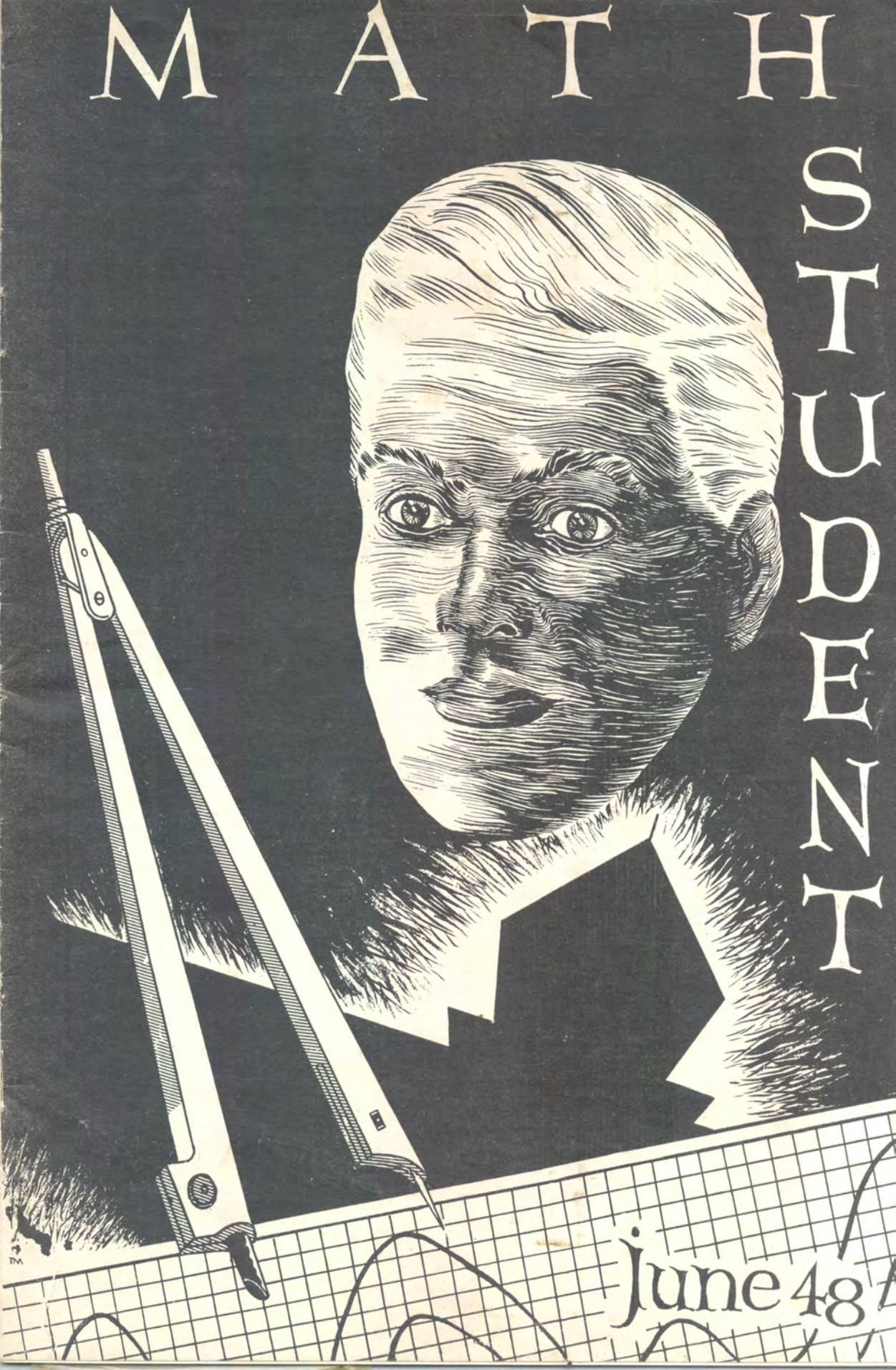
MATH STUDENT



June 48

THE BROOKLYN TECHNICAL HIGH SCHOOL

MATHEMATICS STUDENT

Volume XVII **June, 1948** **Number 1**

In This Issue . . .

Articles

THE PENTAGON ..... *by Israel Kalish* 3
The Pentagon and its relation to the Golden Section

WHEN ALGEBRA FAILS ..... *by Elliot Auerbach* 4
Another system in Mathematics, devised because Algebra and Geometry failed

HOMOTHETIC FIGURES ..... *by Christopher Maximopoulis* 6
An interesting method for Constructions

THE PATH OF LEAST DESCENT ..... *by George Dinsmore* 7
The Cycloid and other curves plus the title

INVERSIONS ..... *by Euval Barrekette* 8
Principles useful in Constructions with Compasses Alone

LITERAL MULTIPLICATION AND DIVISION ..... *by R. Mullin and S. Pottasch* 14

Features and Departments

MATH HONOR ROLL ..... *January*, 1948 10

MATH DEPARTMENT AWARDS ..... *Medalists, January*, 1948 12

CARTOON ..... *by Thomas Malloy and Floyd Arnold* 12

PROBLEM PAGE 13
A puzzle and Literal Examples

MATH ACTIVITIES 15
Clubs, Squad, and Team

THE COVER is from an Original Design by Thomas Malloy. The Back Cover is from photographs by Arthur Sherman.

The Staff

Editorial Board: ELLIOT AUERBACH, *Editor-in-Chief;*
ISRAEL KALISH, *Associate Editor;* THOMAS MALLOY, *Art;*
EUVAL BARREKETTE, FLOYD ARNOLD, GEORGE DINSMORE.

Editorial Assistants: Rolf Lesem, Stuart Pottasch, Raymond Mullin, Barry Jackson, Christopher Maximoupolis, Philip Rosler.

Contributors: Arnold Berger, Burton Berzin, David Butcher, Franklin Kaupp, Marcel Lax, Costas Papaliolios, Robert Rosenberg, Lewis Rosenthal, Aaron Sham, Marvin Stall, Paul Williams.

Photography: Arthur Sherman; *Art*: John Genn, Richard Russell.
Copy Staff: Alvin Zechnowitz (*director*); Ben Fisher, David Gershon, Harry Jacks, Fred Lindheim, Walter Lowenthal, Alan Siegel, Richard Waldman.

Faculty Advisers: Daniel Malament, *Literary;* Louis Sandhusen, *Art;*
William Schwartz, *Business and Finance.*

The *Mathematics Student* is published twice a year, in June and December, by the students of the Brooklyn Technical High School, 29 Fort Greene Place, Brooklyn 1, New York; William Pabst, *Principal;* Julius Freilich, Chairman, *Department of Mathematics.*

The Mathematics Student is a member of the Columbia Scholastic Press Association.

THE PENTAGON

by Israel Kalish

The regular pentagon has many interesting properties, some of which are not readily discernible.

If we draw all the diagonals of a regular pentagon, we form a regular pentagram, the common five pointed star. The diagonals trisect the interior angles of the pentagon. Inside the pentagram, another regular pentagon is formed. If we draw the diagonals of this pentagon, another regular pentagram is formed within which we find another regular pentagon. All angles formed by these operations are simple multiples of 36°.

In the figure thus formed (figure 1), $\angle ABC=108°$, $\angle AFG=72°$, and $\angle GAF=36°$. There are six different sizes of line segments. In order of decreasing magnitude, they are: *AC*, *AB*, *AG*, *FG*, *GM*, and *MN*.

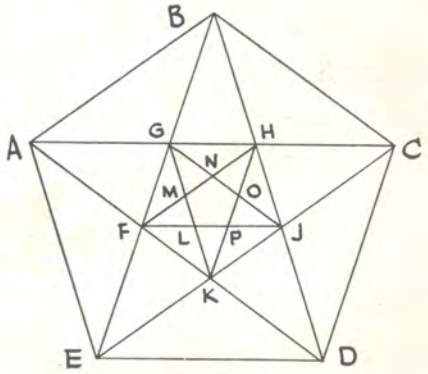


Figure 1

Since all the triangles are isoceles and have equal vertex angles, $\triangle ACD \sim \triangle CHD$; $\triangle CDH \sim \triangle HJC$; $\triangle HJC \sim \triangle HJN$; and $\triangle HJN \sim \triangle NHO$.

Therefore:

$$\frac{AC}{CD}=\frac{CD}{HC}=\frac{HC}{HJ}=\frac{HJ}{NH}=\frac{NH}{NO}$$

From this we see that while the angles are part of an arithmetic progression, the line segments are part of a geometric progression. If we call the common ratio r and the size of the largest line segment a, then:

$$AC = a$$
$$CD = ar$$
$$HC = ar^2$$
$$HJ = ar^3$$
$$NH = ar^4$$
$$NO = ar^5$$

From the diagram we can see that a line segment of the size $AH=BC=CD=ar$ and a line segment of the size $HC=ar^2$ make up a line segment equal to $AC=a$. Therefore:

$$ar^2+ar=a$$
$$r^2+r=1$$

Solving this equation,

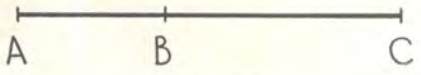


Figure 2

$$r=\frac{-1\pm\sqrt{5}}{2}$$

The positive root of this equation is equal to 0.618033988 . . . This number, $(-1+\sqrt{5})/2$ is an important ratio in geometry known as the *Golden Section*. The Golden Section of a line is that part of a line that is the mean proportional between the entire line segment and the remainder of the line. For instance, in figure 2 if *BC* is the Golden Section of *AC*, then:

$$AC/BC=BC/AB$$

If we consider the length of *AB* to be unity and call *AC* x, then:

$$\frac{1}{x}=\frac{x}{1-x}$$
$$x^2=1-x$$
$$x^2+x=1$$

which is the same equation we had before.

A regular decagon (ten sides) can be inscribed in a circle by using the Golden section of the radius as the length of each side.

Previously we found the value of G to be $(-1+\sqrt{5})/2$.

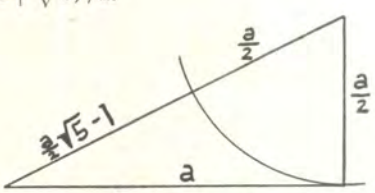


Figure 3

This may be rewritten as $\sqrt{5/4}-1/2$. Therefore, to construct the Golden Section of any line segment a, construct a right triangle whose arms are equal to a and $a/2$ respectively (figure 3). The hypotenuse would therefore be equal to $a\sqrt{5/4}$. From one end of the hypotenuse, mark off $a/2$. The remainder will be equal to $a(\sqrt{5/4}-1/2)$, the G of line a.

WHEN ALGEBRA FAILS

by Elliot Auerbach

In the treatment of equations, the necessity arises for certain operations that can be solved neither by Algebra nor Geometry. A few good examples are; to find the rate of change (or slope of the tangent) of an equation for any value, and to find the area under the curve representing the function. If the equation were in terms of velocity and time, the resulting value for the change would be acceleration. As the value of the rate of change has many other applications, it would be very useful to be able to find it.

In figure 1, AB is a portion of the graph of a function over the intreval between A and B, the change in x is represented by MN, and the change in y by BC. The average change in y with respect to x over the same interval is represented by BC/AC which is the slope of the secant AB. Now, as the change in x (MN) decreases, B approaches A and the secant AB approaches a tangent at A, AD. The exact rate of change in y with respect to that of x is represented by the slope of the tangent, DC/AC.

In doing this type of work, change in x is denoted by Δx, change in y by Δy, so that the value of the rate of change is denoted by $\Delta y/\Delta x$. Since, in finding the exact rate of change, we let the change in x approach zero, the exact rate of change would be

$$\text{limit } \frac{\Delta y}{\Delta x} \qquad \text{as } \Delta x \longrightarrow 0$$

The value of this is called the derivative denoted by

$$\frac{dy.}{dx}$$

For example, let us find the derivative of a simple function, $y=x^2$.

$$y=x^2$$

(the given)

$$y+\Delta y=(x+\Delta x)^2$$

$$y+\Delta y=x^2+2x(\Delta x)+(\Delta x)^2$$

(expanding)

$$\Delta y=2x(\Delta x)+(\Delta x)^2$$

(subtracting the original)

$$\frac{\Delta y}{\Delta x}=2x+\Delta x \qquad \text{(dividing by } \Delta x)$$

Taking the limit as Δx approaches zero, the Δx drops out giving;

$$\frac{dy}{dx}=2x \qquad \text{where } y=x^2$$

We can now find the rate of change for any value of x merely by substituting. Therefore at $x=1$, the rate of change in y with respect to x is 2, for $x=2$, it is 4, and so on.

But using this method, however simple it may seem at first, would be very difficult for an equation like $y=x^{100}$. Just try expanding $(x+\Delta x)^{100}$ and you'll see what I mean.

In order to avoid this, we can develop a set of formulas. The general equation for $y=x^2$ and $y=x^{100}$ is $y=x^n$. Let us begin by finding the derivative of this function. As before:

$$y=x^n$$

$$y+\Delta y=(x+\Delta x)^n$$

(using the binomial expansion)

$$y+\Delta y=x^n+nx^{n-1}(\Delta x)+ \quad \ldots (\Delta x)^n$$

$$\Delta y=nx^{n-1}(\Delta x)+\ldots(\Delta x)^n$$

(subtracting the original)

$$\frac{\Delta y}{\Delta x}=nx^{n-1}+\ldots(\Delta x)^n$$

since all terms on the right except the first contain Δx, they will drop out when the limit as Δx approaches zero is taken and

$$\frac{dy}{dx}=nx^{n-1} \qquad \text{where } y=x^n$$

Using this, the derivative of $y=x^{100}$ can be found very simply. Substituting,

$$\frac{dy}{dx}=100x^{99} \qquad \text{where } y=x^{100}$$

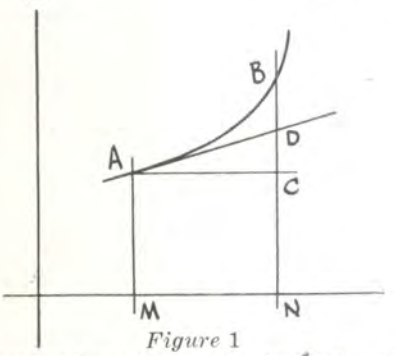


Figure 1

Now we have found the derivative of x^n but what if we had a function of the type $y=3x^3+2x^2+7x+11$. By use of some easily derived formulas, the derivations of which are not given here due to lack of space, the derivative of the whole expression can be found. By one of these formulas, the derivative of $3x^3$ is 3 times the derivative of x^3, or $9x^2$.

Similarly, the derivative of $2x^2$ *is* $4x$, and the derivative of $7x$ is 7. Since 11 is a constant, it does not vary, and therefore its derivative is zero. This may also be obtained by differentiating (the process of taking the derivative) $11x^0$ which is the same as 11. The result here would be $0x^{-1}$ which is equal to zero.

The combined derivatives may be obtained

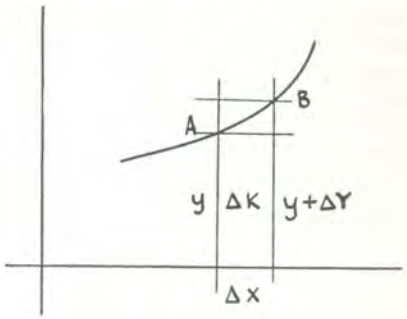


Figure 2

merely by adding. This can also be proven by the Δ method. Therefore:

$$\frac{d}{dx}(3x^3+2x^2+7x+11)=9x^2+4x+7$$

Similarly, many other functions may be differentiated. The value of differentiation and its applications will be discussed later.

As for the second problem, the area under a curve, in figure 2, AB represents a portion of a continuous curve. The area under the curve is represented by ΔK, the change in x by Δx, and that in y by Δy. The area of the larger rectangle is $\Delta x\ (y+\Delta y)$ and the area of the smaller rectangle, $y\Delta x$. Then,

$$\Delta x\,(y+\Delta y) > \Delta K > y\Delta x.$$

As the change in x, Δx, approaches zero, the area of the two rectangles approach the area under the curve. Dividing the above equation by Δx and taking the limit as Δx approaches zero,

$$y+\Delta y > \frac{\Delta K}{\Delta x} > y$$

$$\lim \Delta x \to 0 \quad = \frac{dK}{dx} > y$$

So that to find the area, we must find a value for K, which, when differentiated, gives y. This means that, since we are given the values of y, we must perform an operation which is the reverse of differentiation. This process is called integration, and is indicated by the symbol $\int dx$.

If the given equation were $y=x^2$, then

$$\frac{dK}{dx} = x^2,$$

and by trial, K equals $\frac{1}{3}x^3$

By substituting in this, we can find the area as follows. $\int x^2 dx = \frac{1}{3}x^3$. The area is the difference of the integrals for the two points between which the area is to be found. If the two values of x are 3 and 5,

$$K=\left[\left(\int x^2\,dx\right)_{x=5} - \left(\int x^2\,dx\right)_{x=3}\right]$$

$$K=\tfrac{1}{3}\,5^3 - \tfrac{1}{3}\,3^3$$

$$K=\frac{125}{3}-\frac{27}{3}=\frac{98}{3}=32\,\frac{2}{3}$$

the area under $y=x^2$ *between* $x=5$ and $x=3$.

The integral may also be considered as a summation. In figure 3, area between A and B can be broken down into the rectangular segments shown. The total area of the rectangles approaches the area under the curve when the width of the rectangular segments approaches zero. The sum of the rectangles equals the area under the curve under the above conditions. Therefore, the sum of the rectangles is equal to

$$\int_b^a F(x)\,dx$$

and the integral represents the sum of the elements under the curve.

The above proceses — differentiation and integration — are very useful in the sciences. This can be evidenced by the fact that in the century after their introduction by Newton and Leibnitz (independently), Theoretical Physics advanced more than it

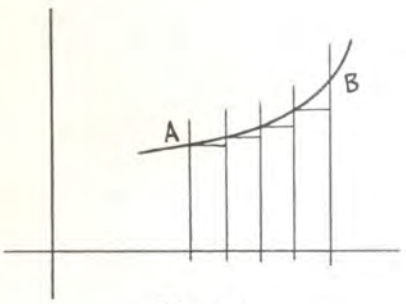


Figure 3

had in all the preceding centuries. The applications of these methods, grouped together under the name, "The Calculus," will be discussed in the second part of this article to be published in the December, 1948 issue.

HOMOTHETIC FIGURES

by Christopher Maximopoulis

Lodged in the corner of a Geometry book is a very interesting but little known subject, Homothetic Figures. "If it's so little known, why not leave it so?", may be the average reaction. Well if you don't read this, no one, not even the Math teacher who made you buy this, will make you read it. However, if you want to learn an easy way to do some difficult constructions, this may enlighten you. The subject, homothetic figures, can be utilized in many constructions, two of which shall be demonstrated here.

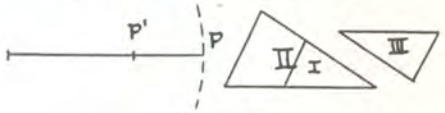


Figure 1

First let us explain exactly what a homothetic figure is. In figure 1a, triangles I, II, and III are similar but only triangles I and II are homothetic (with respect to A). A definition, would be; If A is a fixed point (figure 1b) and P describes a curve and P' is a point on AP points P and P' are said to be homothetic with respect to point A. A is called the Homothetic Center and AP/AP' is called the homothetic ratio or the ratio of similitude. What about the constructions? I know you're dying to see how they're done.

The first construction we shall deal with is the construction of a circle tangent to two interesting lines and passing through a given point, P.

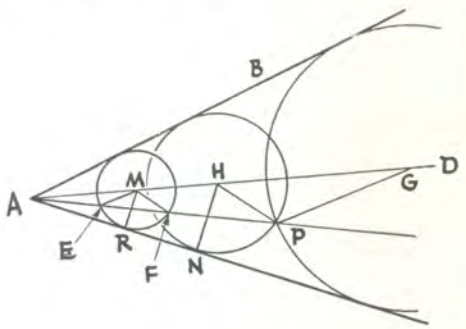


Figure 2

In figure 2, lines AB and AC are the given intersecting lines and point P, the point through which the circle should pass. Since the circle is to be tangent to both lines, the center must lie on the angle bisector. Therefore, first, construct AD the angle bisector of angle BAC.

Next, connect A and P. Then, construct any circle, M, whose center is on AD and is tangent to AB and AC. The circle cuts AP in E and F. Draw MF and ME and draw HP and GP parallel to MF and ME respectively. Points H and G are the centers of the two possible desired circles.

As proof, H is the center of a circle tangent to the lines since it is on the angle bisector. Then, since triangles AMF and AHP are homothetic with respect to A, $AM/MF=AH/HP$, HP is a radius of circle H and point P is on the circle.

In the second construction we shall deal with is inscribing a square in a given triangle. There are many methods but this one

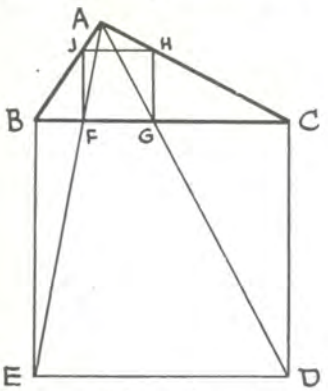


Figure 3

has a certain beauty in its simplicity. The second construction (figure 3) is obtained by constructing square $BCDE$ on the base of the given triangle, ABC. Then draw AE and AD cutting BC at F and G respectively. Finally construct JF and HG perpendicular to FG. Connecting J to H, we have the required square. Proof? Do you really want it? Since the nearest geometry teacher would shoot me if I left it out, I shall state it. The rest of you can just forget about the whole thing, construction included.

Since triangles ABE and AJF and triangles ADC and AHG are homothetic, $JF/BE = AF/AE$ and $AF/AE = FG/ED$, $JF/BE = FG/ED$ or, since $BC=BE$, $JF = FG$. Similarly, $HG=FG$. Therefore $JH \parallel FG$ and, since all sides are equal, the constructed figure is a square.

Many constructions, more difficult to do by other methods, can be easily done by homothetic figures. Can you find some that can be done similarly?

The Path of Least Descent

by George Dinsmore

In 1696, Johannes Bernoulli proposed a problem to his brother Jacob that started a new branch of mathematics known as "The Calculus of Variations." This problem, which has traditionally been called the "Brachistochrone Problem" was:

Suppose that A and B are two points not in the same horizontal plane (figure 1), and suppose that two spheres are released simultaneously at A and allowed to roll from A to B. If the first rolls along a plane and the second along the surface of a concave curve, the second will arrive at B before the first, despite the fact that it has to roll uphill before reaching B and also has to roll a greater distance. It can further be shown that if the plane connecting A and B is replaced by another curve, the curve that shall afford the sphere the quickest descent is a curve known as the *Cycloid.*

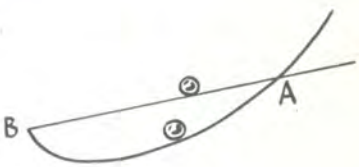


Figure 1

The Cycloid is the curve generated by a point on the circumference of a circle as it rolls along a straight line (figure 2 a). If the point is inside the circle, the curve described is known as a *Curtate Cycloid* (figure 2 b). If a point, P, is taken on the flange of a locomotive wheel rolling on a rail (figure 2 c), a curve known as the *Prolate Cycloid* is generated (curve AB). Strangely enough, if the locomotive is moving forward, there is one place on the locomotive that is moving backward, the flange, a point on which describes the little loop at the bottom of the Prolate Cycloid.

The Cycloid has two interesting properties attached to it. The first is that the length of a single arch is equal to four times the length of the generating circle. The other is that the area under one arch is equal to three times the area of the generating circle.

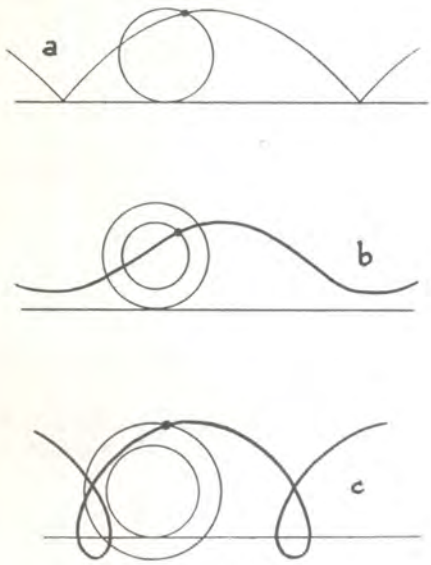


Figure 2

Another type of Cycloid is the *Hypocycloid.* The Hypocycloid is the curve generated by a point on the circumference of a circle rolling around the interior of a larger, fixed circle. The most notable of these Hypocycloids is the *Astroid,* or Hypocycloid of four cusps (figure 3 a). The Astroid is the curve generated by a circle rolling around the interior of another circle having four times the diameter of the first. The general equation for the graphic representation of the Astroid is $x^{2/3} + y^{2/3} = a^{2/3}$ in which a is the diameter of the larger circle.

An interesting property of the Astroid is

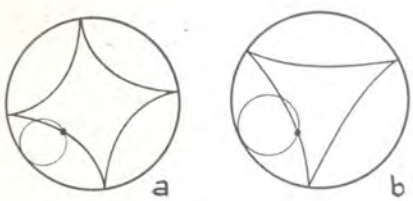


Figure 3

that the area of the curve sided figure in the center is equal to twice the area of the small generating circle.

(*Continued on Page* 9)

INVERSIONS

by Euval Barrekette

Many problems in construction can be solved or simplified by means of "geometrical transformations" or "inversions." By definition, the inverse of a point is another point on the line joining the first with the center of any circle, so that the product of the distances of these points from the center is equal to the radius squared. In figure 1, P is the inverse of P' and P' the inverse of P, if $OP \times OP' = r^2$.

The following are the four fundamental inversions:

a) The inverse of a line through the center of a circle is a line through the center of the same circle.

b) The inverse of a line not through the center of a circle is a circle through the center of the first circle.

c) The inverse of a circle through the center of a circle is a line not through the center of the second circle.

d) The inverse of a circle not through the center of a circle is another circle not through the center of the circle.

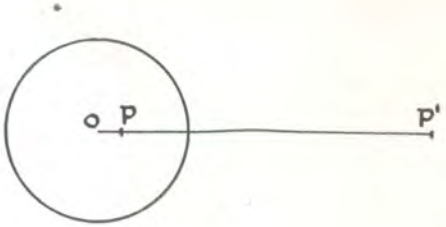


Figure 1

In *a* we have a circle with a line through its center (figure 1). The inverse of the line is the locus of the inversions of all its points. The inverse of any point within the circle, on the line, is a point outside the circle on the same line. Also, the inverse of any point outside the circle, on the line, is a point within the circle on the same line. Therefore the inverse of the line is the same line with its points interchanged.

In *b* we have a circle O and a line AB not through O, (figure 2). From O we drop $OC \perp AB$. Now we join any point P on AB with the center and find the inverse points, C' and P', of C and P respectively. From the definition of inversion we obtain the following relationship:

$$OC \times OC' = r^2 = OP \times OP'$$

$$OC'/OP' = OP/OC$$

Therefore $\triangle OP'C' \sim \triangle OCP$ for they have one angle in common and the including sides are proportional. Thus $\angle OP'C' = \angle OCP =$ right angle.

As point P moves along AB point P' moves within circle O, and $\angle OP'C'$ will remain a right angle. Therefore, the locus of

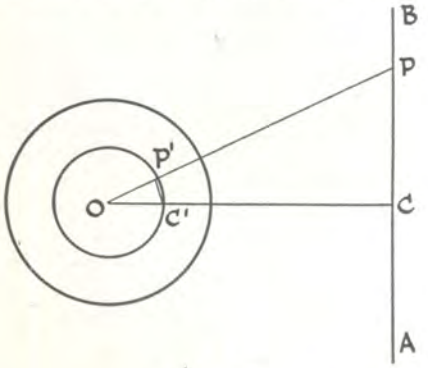


Figure 2

P' will be a circle having OC' as its diameter.

In *c* we have a circle through O (figure 2). If we reverse the proof of *b* we see that the inverse of this circle is the line AB not through O.

In *d* we have a circle O and another circle M not through O (figure 3). To find the inverse of circle M we draw a tangent OC to circle M, a secant OAB to circle M, and find the inverse points, B' and A', of B and A respectively. From the definition of inversion we get:

$$OA \times OA' = OB \times OB' = r^2$$

also: $OA \times OB = OC^2$ for the product of a

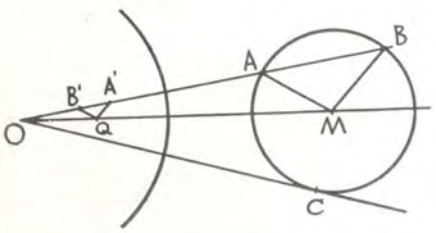


Figure 3

secant and its external segment is equal to the square of the tangent. Dividing we get:

$$\frac{OA \times OA'}{OA \times OB} = \frac{OB \times OB'}{OA \times OB} = \frac{r^2}{(OC)^2}$$

$$\frac{OA'}{OB} = \frac{OB'}{OA} = \frac{r^2}{(OC)^2}$$

Since the radius and the tangent are constant $r^2/(OC)^2$ is the same for all positions of OAB. Now through A' draw $A'Q$ parallel to BM. In triangles $OA'Q$ and OBM we get:

$$OQ/OM = OA'/OB = A'Q/BM = r^2/OC^2$$

OM, BM, and r^2/OC^2 are constant, so that OQ and $A'Q$ will remain the same for all positions of secant OAB.

Draw $B'Q$ and AM.

Since $\frac{OB'}{OA} = \frac{OA'}{OB} = \frac{OQ}{OM}$

$B'Q \parallel AM$ and $\triangle OB'Q \sim \triangle OAM$

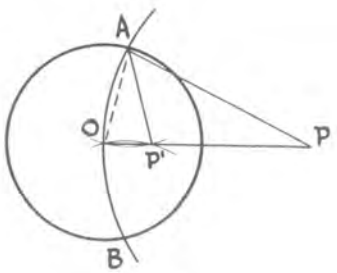


Figure 4

Therefore $\frac{B'Q}{AM} = \frac{OQ}{OM} = \frac{A'Q}{BM}$

Now, since $AM = BM$, $B'Q = A'Q$

This means that no matter what position secant OAB assumes, the values of $A'Q$ and $B'Q$ will not change, and the position of point Q will remain the same. Under these circumstances the locus of points A' and B' is the circle whose center is Q and whose radius is $A'Q$.

These inversions can be constructed with compasses alone, since the inverse of a point can be thus constructed.

In figure 4 we see how to locate P' the inverse of P with the aid of compasses only, by drawing an arc through O whose center is P and whose radius is OP, cutting circle O at A and B. With A and B as centers and OA as our radius we describe arcs OP'. We thus obtain triangles OAP' and OAP which are isosceles and similar because of common angle O. Therefore,

$OP'/OA = OA/OP$ or $OP' \times OP = OA^2 = r^2$

and P' is the required inverse point of P.

It is possible by means of inversion, to construct with compasses alone anything that can be constructed with a straight edge and compasses.

The Path of Least Descent

(*Continued from Page* 7)

Another Hypocycloid is the Hypocycloid of three cusps. This is the curve generated by a point on the circumference of a circle rolling around the interior of another circle whose diameter is equal to three times that of the small generating circle (figure 3 a).

The last of this series of Hypocycloids is one in which the diameter of the larger circle is equal to twice the diameter of the smaller circle. This curve is perhaps the most unusual for it is a diameter of the larger circle (figure 4).

The Hypocycloid is useful as well as interesting. If a bar is attached to the smaller circle, as the circle rotates the bar will move back and forth along the diameter. As it moves, so will the bar. Circular motion is thus transformed into reciprocating motion.

This principle is used in many machines and is indeed a unique one. In the operation of a shaper the rotary motion of the

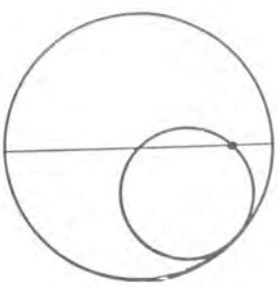


Figure 4

gear is transformed to the reciprocating motion of the ram. Even greater uses of this principle are made in modern rapid production machines.

HONOR ROLL

Every term the Mathematics Department awards Certificates of Excellence to the boys distinguishing themselves in the term's work. In the non-Regents classes a minimum of 85 per cent in the final examination and at least 85 per cent as the term mark, is required for Certificates of Merit. In the Regents classes a minimum of 95 per cent on both the Regents examination and term mark are required. The Mathematics Honor Roll, as determined by pupils obtaining at least 95 per cent for the term and 95 per cent on the final or Regents examinations follows:

MATHEMATICS I

PURE AND APPLIED

Name	Class	Term Mark	Exam. Mark
Adams, S.	A12	95	98
Apfelbaum, C.	A12	95	95
Apostol, S.	B13	95	100
Ashford, A.	E11	95	100
Austenberg, K.	C11	97	100
Bagnasco, K.	C12	95	98
Barry, R.	F13	95	95
Bassi, A.	A12	98	100
Baum, T.	C11	97	100
Beckendorf, P.	B13	95	98
Binkow, M.	B12	98	100
Braverman, P.	A12	98	100
Bros, L.	A12	95	98
Butensky, A.	F12	95	100
Cernasia, T.	F13	98	98
Chamberlain, W.	A12	98	100
Champlin, L.	C11	95	98
Chinowski, S.	B11	98	100
Ciecierski, W.	A12	98	100
Cohen, S.	C11	96	95
Commerford, G.	B11	95	100
Cosentino, L.	F12	95	100
Davidson, G.	C11	97	100
Dorner, G.	C12	96	98
Downing, K.	B11	97	100
Eberle, R.	C11	97	100
Eccles, J.	D14	95	98
Eisenberg, B.	F13	96	98
Ernandees, F.	C11	97	100
Ettari, P.	B22	95	98
Fazio, J.	B11	98	100
Files, C.	A12	95	100
Fox, A.	F12	95	100
Freudenberg, R.	C12	95	95
Fruchtbaum, H.	A11	98	100
Genderman, R.	A13	96	98
Glass, H.	A11	95	100
Goldbart, L.	C11	95	100
Golub, J.	A11	95	96
Grassman, H.	F11	95	100
Hagen, K.	A12	95	96
Hansen, W.	C12	95	100
Henry, R.	D13	95	95
Hershenson, M.	B13	98	100
Hight, N.	D14	95	96
Holderman, F.	F11	95	95
Jung, W.	A12	95	100
Kaiser, F.	D13	96	100
Kantor, D.	B14	95	95
Katims, R.	C11	95	95
King, L.	C11	95	98
Kiadko, M.	C51	97	98
Klein, R.	F13	96	100
Kogan, S.	D12	95	100
Kohrs, R.	A11	95	100
Konlande, J.	A12	95	98
Kosofsky, J.	B12	95	100
Kress, R.	D13	95	100
Kreutinger, F.	F13	95	100
Lainas, G.	F11	95	98
Lehmann, O.	E13	95	96
Leighley, R.	E12	95	100
Logan, R.	F12	95	100
McConnell, J.	A12	98	100
McKee, W.	A12	95	100
Marott, R.	D12	95	98
Massa, R.	A12	98	100
Mollicone, R.	D11	97	100
Morris, M.	A13	95	100
Moskowitz, S.	C13	96	98
Muller, R.	E12	98	98
Naley, J.	C14	95	100
Nielsen, P.	A12	98	98
O'Connor, M.	E12	95	100
Olson, R.	C11	95	100
Petrany, R.	A12	95	95
Plotnick, W.	D14	95	100
Rapp, D.	E13	95	98
Reda, F.	A12	95	100
Ritsi, G.	D13	96	100
Rosenbaum, R.	F13	100	100
Rosenberg, D.	C12	95	100
Rosenberg, S.	B11	98	100
Salemi, N.	B12	95	100
Scheiblich, R.	D14	95	100
Seckendorf, P.	B13	95	98
Segal, A.	A12	98	100
Shapiro, A.	F11	95	100
Sheil, J.	D14	95	100
Sirch, R.	C11	95	100
Solomon, A.	B12	98	100
Straub, E.	F12	98	100
Strickholm, O.	C12	97	100
Swain, J.	E12	95	98
Toman, D.	E11	98	100
Trachtman, L.	E12	95	95
Trenk, A.	F12	95	98
Tricarico, R.	F11	95	100
Van Praag, D.	D12	95	100
Verrios, A.	C12	98	100
Warshay, M.	F13	95	95
Wiegand, A.	B11	95	100
Weinstein, B.	C12	95	95
Weinstein, M.	A12	100	100
Wittenstein, J.	E11	95	98
Zalman, G.	B11	95	100

MATHEMATICS II

PURE AND APPLIED

Ansoorian, M.	C22	95	99
Becker, R.	B23	98	98
Della-Torre, E.	F23	95	95
Dorato, P.	D21	95	95
Duffy, J.	F21	95	96
Eggen, A.	A22	95	95
Foti, S.	E22	95	98
Freeman, J.	B22	95	97
Goldstaub, J.	A22	100	100
Griemsmann, W.	A22	95	97
Hergenrother, W.	C23	95	96
Joseph, R.	F23	95	95
Kaesz, H.	E23	95	98
Kittiver, L.	C23	97	95
Krutchik, A.	E23	95	97
Kulgein, N.	E23	95	98
Lanstein, R.	A22	100	100
Leccese, F.	D22	98	99
Lee, D.	A23	95	100
Levine, P.	A23	95	95
Lieberman, P.	A23	100	100
Maffei, A.	E23	95	97
Maurer, S.	A21	98	98
Meyers, R.	B22	95	95
Moskowitz, S.	E23	95	98
Nebocate, G.	F23	95	95
Nemet, R.	A22	100	100
Nespole, A.	A23	95	95
Pearson, A.	A22	95	98
Puttre, P.	A22	95	100
Reichman, S.	E23	95	99
Richards, E.	A22	95	98
Riemenschneider, W.	A22	95	98
Russell, T.	F21	96	99
Stein, P.	F21	98	100
Thollot, P.	A23	95	95
Trabold, A.	E22	95	97
Vopat, R.	A23	95	95
Weber, W.	B22	96	97
Weinstein, L.	A21	98	98
White, E.	B21	95	99
Zahn, R.	A23	100	100
Zeitlin, P.	A22	95	100

MATHEMATICS III

PLANE GEOMETRY

Bennett, A.	B31	95	100
Bernstein, H.	B31	96	99
Bunimowitz, I.	F33	95	100
Caradec, A.	F22	95	95
Cheng, H.	C21	95	99
Cohen, A.	B32	95	98
Dinsmore, G.	C31	98	100
Drogin, E.	B31	96	98
Dunkelman, P.	A32	95	95
Farinella, S.	C21	95	97
Foti, C.	B33	95	100
Geduld, M.	B33	95	100
Gilbert, P.	A31	96	99
Hart, J.	F31	96	98
Harary, C.	D32	96	96
Hess, J.	F32	96	97
Jorgensen, H.	B33	95	97
Kaplan, A.	F32	96	98
Kluger, J.	A32	99	99
Koch, C.	B32	95	100
Korobow, L.	F31	97	100
Lande, K.	F33	99	100
Lignos, L.	C41	95	99
Lipow, B.	B33	95	99
Loheac, H.	B31	95	99
Loscalzo, D.	B32	95	98
Makara, J.	C32	95	100
Martin, G.	A31	96	96
Mills, M.	F31	97	100
Morrone, J.	A31	95	100
Muhlker, W.	E32	98	100
Muschler, W.	F33	95	100
Oran, F.	F31	96	98
Paul, R.	F31	96	98
Plotsky, G.	B32	95	100
Quottrochi, J.	A31	96	97
Rabinowitz, M.	F32	95	95
Reiner, F.	F33	96	100
Reuter, W.	D32	98	100
Rieke, W.	F32	95	100
Rizzuto, J.	A31	96	100
Robisch, R.	F31	97	100
Sacks, G.	A31	96	100
Schnopper, H.	F32	99	98
Serrand, R.	D33	99	100
Smith, J.	B32	98	100
Steiner, P.	F32	95	100
Thompson, R.	F33	95	96
Tupper, J.	E31	95	100
Wasserman, E.	A31	97	95
Zeifman, L.	B53	96	98

MATHEMATICS IV

PLANE GEOMETRY, REGENTS

Altonji, G.	B43	96	98
Ames, M.	B43	99	99
Barone, S.	D41	97	97
Bashe, R.	B41	96	99
Bazik, J.	A42	95	96
Beckman, R.	C42	98	100
Berkson, W.	F43	95	97
Bernstein, S.	F41	98	100
Blair, I.	C43	95	95
Blair, W.	A41	95	97
Botshon, M.	F41	97	97
Cardinale, D.	C41	98	100
Chomowig, F.	A41	95	97
Cohen, B.	F43	98	100

Colborn, G.	F41	98	99
Coleman, M.	E42	95	100
Connors, T.	B43	95	100
Cupo, W.	A41	95	98
Danz, W.	F43	96	98
Dinhoffer, A.	B43	99	100
Dolmatch, E.	B43	97	97
Elenko, R.	B41	96	98
Epstein, M.	B43	95	97
Erickson, R.	C43	95	98
Fader, I.	B43	95	96
Feldman, R.	B41	95	98
Felt, J.	A41	98	98
Finley, R.	D42	95	97
Frank, A.	B41	97	99
Fredericks, G.	E42	95	100
Freiman, C.	A42	99	99
Garson, N.	E41	95	97
Gloates, E.	B41	95	96
Goodlet, J.	D43	95	95
Gottlieb, L.	D41	95	97
Grossberg, A.	B42	96	98
Gruverman, I.	D41	99	100
Hartman, M.	B43	96	95
Hauben, R.	A42	98	96
Heyner, R.	D43	95	95
Homsey, M.	B43	99	100
Johnson, M.	A41	98	98
Judd, S.	E41	95	98
Keissch, B.	C51	97	97
Kohl, W.	B52	96	96
Kraus, B.	F41	96	98
Lapar, R.	C43	95	95
Lazovick, L.	D41	95	98
Levine, J.	B43	99	98
Levitt, S.	E41	95	100
Link, R.	B42	97	99
Loebel, M.	D41	95	96
Loewenstein, H.	B42	97	100
Mackenzie, R.	A41	97	95
Mahler, C.	C43	95	95
Maieli, V.	B42	95	98
Manzo, F.	F41	97	100
Meberg, A.	E41	95	100
Merz, E.	F43	95	96
Moccia, M.	C43	97	96
Morea, S.	F43	98	99
Moreno, F.	A41	95	97
Nicolai, D.	A41	95	98
Oliva, V.	F41	95	95
Papageorgian, M.	E42	95	98
Parody, C.	C42	97	98
Pass, I.	D43	95	98
Paticoff, H.	A41	95	95
Pederseu, C.	F43	98	98
Peterson, R.	A41	95	96
Piper, W.	A41	95	95
Preschlack, J.	F41	98	100
Rosenzweig, M.	B41	95	99
Rosler, L.	F41	99	100
Rubenfeld, H.	F41	96	96
Rubin, I.	D43	95	100
Schindler, R.	E41	96	97
Schindler, S.	A41	95	96
Schwartzberg, B.	A42	95	96
Schwinger, A.	A42	95	100
Seigal, A.	B41	97	100
Shapiro, H.	D42	98	99
Shor, I.	B41	97	100
Shukaitis, V.	E41	96	95
Silverfine, L.	C42	95	98
Slavin, J.	D43	95	95
Specter, C.	F42	99	100
Thollot, P.	A23	95	95
Voss, J.	B21	96	100
Wasserman, B.	F41	95	95
Weinstein, H.	B43	98	97
Wern, D.	A41	95	95
Whiting, R.	B53	96	98
Wiederhorn, R.	F41	96	96
Wood, J.	B42	97	100

MATHEMATICS V

Intermediate Algebra, Regents

Axelrod, M.	F51	98	100
Alfiero, S.	F52	95	98
Alaimo, S.	D52	95	96
Barbieri, J.	E53	95	98
Berger, A.	F52	95	99
Blasnick, W.	F53	95	95
Chiarella, A.	E53	97	97
Chin, J.	A52	98	98
Crader, C.	F52	98	100
D'Aytoriyo, G.	A52	98	98
Ebenfield, H.	B51	99	98
Ehrman, L.	B53	99	100
Fleiss, P.	A52	96	96
Freedman, M.	F51	95	98
Gerber, E.	F53	98	98
Gold, R.	F53	97	95
Grossgold, M.	B53	100	100
Heller, S.	F53	97	98
Helpand, L.	B51	95	95
Hunt, J.	A53	95	95
Isaacs, H.	A52	100	100
Kaestle, J.	B51	99	100
Kahrs, P.	A52	95	98
Kaufman, A.	F51	98	100
Kelly, W.	B51	98	96
Kladko, M.	C51	97	98
Kozar, M.	B52	95	97
Kranser, L.	F51	98	98
Kreshin, L.	F51	98	100
Lerner, B.	A52	98	98
Levine, E.	B52	97	98
Lieberg, R.	B51	98	97
Lindner, R.	B51	99	100
Lowey, H.	D53	96	99
Lopez, A.	F51	95	99
Marcus, I.	A53	95	96
Markowitz, S.	C52	95	96
Mayrbaurl, R.	C53	97	100
McGoigon, J.	C51	96	95
Miller, W.	E53	98	99
Nashman, M.	B53	95	100
Nelson, C.	E51	95	95
Nervich, J.	A52	95	96
Offenberg, M.	F51	98	98
Papaliolios, C.	F52	99	100
Pastor, A.	A52	98	98
Peters, J.	C52	95	96
Romani, F.	F53	99	98
Rumzi, K.	E51	95	95
Runiewicz, W.	B53	100	100
Santoro, A.	C52	98	100
Schwartz, H.	C52	95	98
Schwartz, L.	A52	95	96
Sender, L.	C53	95	96
Seroka, S.	F53	95	100
Shepheard, R.	B53	95	97
Signore, R.	B52	98	98
Stratakos, T.	F51	95	96
Strohbeck, C.	B51	99	96
Skowronek, R.	B51	97	95
Vaillant, A.	E53	97	97
Werner, H.	A52	95	96
Wickstrom, G.	F52	98	100
Witt, F.	C52	95	96
Witt, L.	D52	95	100
Wolfeld, R.	B51	99	96
Zerfman, L.	B53	96	98

MATHEMATICS VI-R

Trigonometry, Regents

Abramson, M.	F61	95	99
Auerbach, E.	B62	98	100
Brady, J.	D62	97	100
Bodenstab, C.	A63	95	95
Casals, A.	A63	96	99
Chin, H.	A63	95	95
DeMair, R.	D61	97	98
Drucker, D.	A63	98	100
Duffy, G.	D61	95	97
Ehnhuus, R.	A63	95	96
Ehrlich, R.	C62	99	100
Favre, J.	F63	98	95
Furst, S.	A63	95	95
Gartner, J.	A63	95	98
Gordon, I.	A63	96	100
Greenberg, J.	F61	99	98
Greenspan, K.	A63	95	95
Guba, A.	A63	96	100
Hathaway, R.	F63	98	98
Horden, M.	F62	95	100
Horowitz, J.	B61	98	100
Janofsky, L.	A61	95	98
Johnson, R.	D62	97	100
Kalish, I.	B61	95	100
Katz, E.	F61	95	98
Kaufman, A.	F51	99	100
Kinard, J.	D61	97	100
Klem, W.	F63	98	100
Knauer, M.	B61	95	100
Kurzius, S.	B61	95	96
Lesem, R.	A63	96	99
Lokkeberg, R.	B61	95	100
Lopez, A.	A63	96	99
Mardoc, V.	B61	95	100
Mazzola, L.	B62	96	98
Newman, A.	B61	95	100
Nilsson, W.	B62	95	98
Pattasch, S.	F61	99	100
Pedowitz, I.	F61	95	96
Perlmutter, D.	B62	98	100
Rear, S.	C62	99	100
Reuther, D.	B62	96	100
Ricci, R.	B61	95	100
Santoriello, S.	B61	97	98
Sargent, A.	A63	95	95
Seidman, J.	B62	95	100
Silbermann, F.	A63	95	95
Simon, L.	A63	99	100
Simonelli, N.	F63	100	100
Steinberg, A.	F61	99	99
Sternberg, H.	A63	96	98
Svensson, L.	D61	97	100
Wapnick, N.	A63	95	97
Weill, D.	B62	95	100
Wiff, P.	A63	95	98
Zechnowitz, A.	B61	98	100
Zurl, A.	A63	99	100

MATHEMATICS VI-T

Technical Trigonometry

Asbell, W.	E62	95	97
Briglio, J.	A61	95	95
Capasso, M.	A61	95	100
Casteran, R.	E63	95	95
Cranstoun, D.	E61	95	97
Dilpare, A.	E62	97	99
Ek, H.	E62	95	96
Johnson, A.	A61	95	96
Kwawer, A.	E63	95	97
Lignau, W.	D61	96	99
Liskamm, W.	A61	95	95
Nelson, A.	E63	95	100
Nietert, R.	E62	96	98
O'Halloran, H.	D61	95	95
Raspante, P.	A61	95	97
Sigman, T.	C61	95	100
Smith, E.	C61	95	96

MATHEMATICS VII

Solid Geometry, Regents

Abraham, D.	A71	97	100
Altschaeffl, A.	E71	98	97
Anderson, B.	E83	95	98
Apfel, A.	A73	95	98
Aublien, H.	C72	97	98
Barrekette, E.	C73	100	100
Beninati, S.	C72	95	100
Berezuk, F.	E71	95	99
Bessen, A.	B62	99	100
Bierman, R.	F72	99	100
Birnbaum, R.	C71	98	100
Blanda, L.	F82	95	95
Cagna, B.	F72	95	98
Chajes, A.	F72	95	96
Curwin, R.	B72	98	100
D'antonio, A.	B52	95	95
Donnelly, J.	B72	95	98
Dunbar, D.	C71	95	95
Durchlaub, E.	E71	95	98
Erlichson, H.	F72	98	100
Fanelli, R.	F72	99	100
Finkelstein, A.	F72	95	95
Foley, J.	C71	99	100
Fogiel, M.	A71	95	95
Freedman, S.	A71	95	99
Freeman, A.	B71	97	100
Fried, S.	C72	97	98
Friedensohn, G.	A71	97	100
Gaines, S.	A72	98	100
Goldman, L.	F72	95	95
Green, M.	B71	95	95
Hahn, G.	B71	95	95
Hahn, L.	C71	95	97
Heitman, R.	C72	96	98
Hoercher, H.	B72	96	99
Honigsberg, C.	B71	95	100
Hublien, H.	C72	97	98
Kane, J.	A71	96	99
Kantor, F.	B71	95	97
Kantor, I.	C71	95	98
Kaplan, L.	B71	98	100
Kateb, G.	B71	95	97
Kinsey, G.	B71	95	100
Kiustljuger, J.	B71	98	100
Korn, A.	E71	98	98
Krieger, L.	D71	95	98
Kristiansen, K.	B71	96	98
Kypar, M.	C71	97	98
Lax, M.	A71	98	100
Lehrer, J.	C71	97	97
Leibowitz, B.	C71	96	95
Lembeck, W.	B71	97	98

Lifsey, J.	C71	96	96
Melnick, J.	F72	95	95
Millman, E.	F72	98	98
Narlin, J.	B81	98	100
Ortiz, A.	F63	95	99
Parton, A.	A71	95	95
Perla, G.	B71	95	98
Perry, L.	A71	95	98
Peterec, R.	A73	95	97
Popofsky, M.	C71	97	97
Povman, M.	B71	99	100
Reisacher, W.	B72	95	97
Rockwell, R.	D71	95	98
Roeder, A.	C73	95	97
Rosen, R.	A73	98	98
Russ, W.	B72	95	96
Sadowski, H.	B72	95	99
Schapiro, B.	B71	98	100
Salter, E.	B72	95	95
Samson,R.	A73	95	95
Shapiro, L.	A73	95	97
Silverberg, I.	F72	96	98
Sobol, H.	D71	99	100
Stein, M.	F72	96	96
Stern, H.	F71	97	100
Stimpel, R.	B72	95	96
Thompson, F.	C71	99	99
Weber, J.	B71	95	99
Weinberger, J.	F72	97	95
Weitzner, S.	F72	95	95
Wichern, C.	E71	95	95
Willerns, C.	D71	97	97
Wishnia, A.	A71	99	100
Zevin, M.	A73	96	97
Zuckerbrod, M.	A71	95	95
Zuckerman, R.	A73	96	98
Levine, R.	C72	95	100

MATHEMATICS VIII

ADVANCED ALGEBRA, REGENTS

Ambs, H.	B82	97	99
Anderson, W.	B81	95	100
Annunziato, A.	D73	95	98
Aranowitz, G.	F82	97	98
Ausfresser, D.	F82	97	99
Bachman, H.	C81	95	98
Bagner, G.	F81	97	98
Beran, M.	F81	96	96
Bron, W.	A81	95	98
Caplan, M.	B82	95	98
Carlin, R.	C81	97	99
Cohen, S.	T81	97	98
Epstein, M.	B81	95	98
Frank, J.	A81	96	98
Freidstein, J.	B82	95	99
Friedman, M.	A82	98	100
Finkernagel, R.	F82	95	100
Gallant, D.	F82	95	100
Gelertner, H.	F82	95	100
Greenberg, H.	A81	97	98
Griwatz, B.	B81	95	100
Gorstein, F.	A81	95	95
Hirshfeld, A.	F81	100	100
Johnson, W.	A82	96	100
Kalish, S.	B81	95	100
Karl, W.	A81	97	97
Karlin, J.	B81	98	100
Kaplan, I.	B82	95	98
Keilsohn, J.	B81	95	95
Keller, R.	C81	95	99
Klauser, F.	C81	95	98
Kostiw, N.	A82	98	96
Krasner, M.	C73	95	99
Lamendola, C.	B81	95	100
Lang, R.	F81	96	96
Lerner, A.	F81	97	98
Levine, J.	B81	95	98
Levy, R.	F82	95	95
Lundenberg, A.	F82	95	99
Markowitz, G.	B81	95	97
Massa, H.	A81	95	97
Mendlesohn, M.	F82	95	98
Metzlen, C.	T81	97	100
Meyer, R.	C82	95	100
Milder, M.	F82	95	98
Michton, L.	A82	95	100
Monkmeyer, P.	A82	99	100
Neumaier, A.	A82	96	98
Olin, M.	F81	98	100
Pascale, D.	F81	95	98
Perlman, N.	B82	95	100
Puca, G.	F82	98	100
Raphael, M.	B81	98	100
Reff, T.	F81	100	100
Reiss, J.	B81	95	100
Rey, R.	A81	95	100
Rosenblatt, J.	A82	98	100
Ross, D.	A82	98	100
Salz, H.	A81	100	100
Saperstein, A.	B81	95	98
Schwaninger, J.	F81	95	100
Schwartz, S.	B82	95	100
Seelig, B.	B81	95	100
Smith, J.	F81	95	100
Smulow, J.	F82	95	95
Sperling, P.	A82	98	97
Steinberg, H.	A82	97	97
Steiner, H.	B82	95	99
Stengel, F.	C81	95	100
Stromsen, E.	E82	95	100
Sturiale, P.	D73	95	100
Szymber, J.	F82	95	96
Ton, J.	E83	95	99
Voigt, R.	C81	95	100
Weinstat, H.	B81	97	100
Wrieden, E.	A81	95	98
Yabroudy, G.	F82	95	95

Department of Mathematics Medal Awards

JANUARY, 1948

The Mathematics Department awarded medals to the top ranking graduating students from the College Preparatory Course and the Terminal Courses. In the College Preparatory Course medals were awarded on the basis of Regents marks (Plane Geometry, Intermediate Algebra, Trigonometry, Solid Geometry, and Advanced Algebra). In the Terminal Courses, medals were awarded on the basis of term marks.

MATHEMATICS FOUR YEARS

GEORGE HIRSCH—*Gold Medal*
ALAN HIRSHFELD—*Gold Medal*
Bronze Medals

Raymond Carlin
Herbert Weinstat
Herbert Gelernter
Carl Johnson
Jay Carlin
Miles Friedman
William Kleinman
Charles Metzler
Harvey Salz
Robert Levy
Jules Rosenblatt
Melvin Epstein
Joseph Frank
Peter Monkmeyer
Alvin Saperstein
Charles Wenklehaus
Robert Finkernagel
Donald Pascale
Stanley Storfer
Martin Raphael
Arthur Cimento
Charles Lamenfola
David Gallant
Stanley Schwartz
Gerald Aronowitz
Martin Metzger
Jay Michton
Morton Milden
Morton Olin
Burton Seelig
Herbert Steinberg
Frederick Stengel

MATHEMATICS THREE YEARS

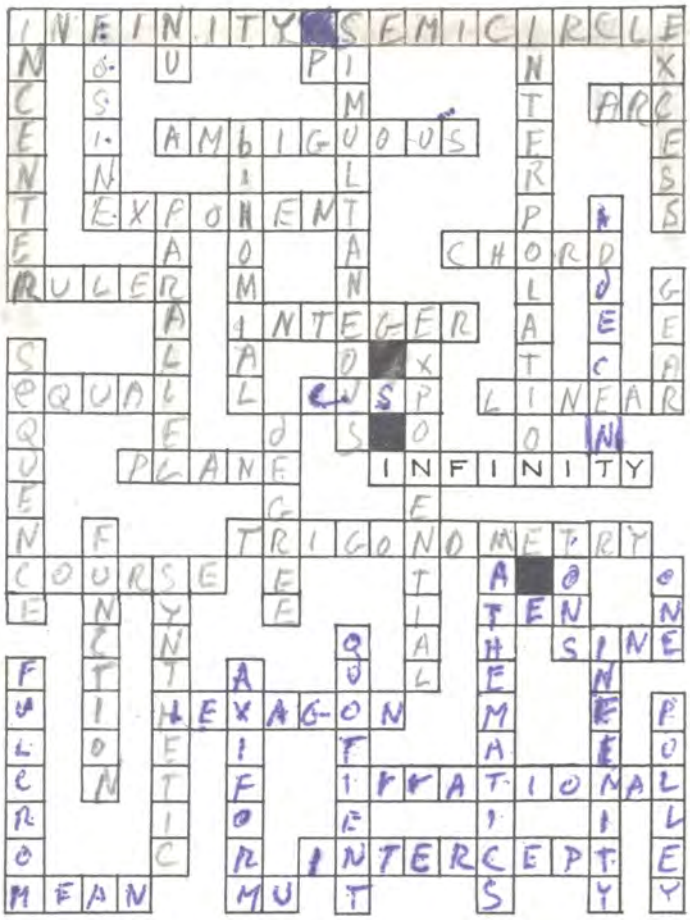
MARTIN ERLEBACHER—*Gold Medal*
Bronze Medals

Stephen Levy
John Brueger
Victor Horlick
Steve Bloom
Paul Reitman
George Kaufer
John Gouvier
Kirk Kazarian
Walter Weiss
Roland Johnson
Walter Carlsen

ENCORE!

PROBLEM PAGE

In the appropriate spaces in the puzzle at the left place the words listed below, so that they interlock and complete the puzzle.



Two letter words;
Mu
Nu
Pi

Three letter words;
arc
one
ten

Four letter words;
cusp
sine
mean
tons
gear

Five letter words;
plane
equal
ruler
chord

Six letter words;
linear
pulley
degree
cosine
course
excess

Seven letter words;
axiform
fulcrum
hexagon
integer

Eight letter words;
~~incenter~~
~~exponent~~
~~infinity~~
function
sequence
parallel
binomial
quotient
adjacent
infinite

Nine letter words;
ambiguous
synthetic
intercept

Ten letter words;
irrational
semicircle

Eleven letter words;
mathematics
exponential

Twelve letter words;
simultaneous
trigonometry

Thirteen letter words;
interpolation

—A. BERGER
C. PAPALIOLIOS

Literal Examples

(Solution on Page 15)

Can you solve problems in literal multiplication? After reading the article on Page 14 you should be able to solve the following:

```
   HFG
   BCD
  ----
  EFFD
  EHFG
 AFFB
------
AEBBCD
```

```
     FGBB
  ---------
DCF)DCADAFA
     FGD
    ----
     FCA
     GAE
    -----
      BHF
      BEA
     -----
      BEA
      BEA
     -----
```

Literal Multiplication and Division

by S. Pottasch and R. Mullin

Although mathematics may seem like nothing more than homework and tests, it does have a lighter side in the form of number games and puzzles. Literal multiplication and division problems not only contain multiplication and division but addition and subtraction as well. Here are two typical problems. Can you solve them?

	I	II	III	IV	V	VI	VII
1				E	B	C	E
2					S	R	L
3				C	N	D	L
4		E	L	L	G	R	
5	E	N	S	E	S		
6	E	E	V	S	L	G	L

	I	II	III		IV	V	VI	VII	VIII	IX
1							D	D	G	
2	C	F	C	)	C	B	B	H	Y	G
3					E	B	D			
4						E	Y	Y		
5						E	B	D		
6							Y	H	G	
7							Y	H	G	

Let us begin with the multiplication problem. We see that L, R, and S, in line 2 when multiplied by E give us L, R, and S, respectively. Therefore E must be 1 or 5. If it is 5, S, R, and L must be all odd numbers. When E in IV line 1, is multiplied by S, R, and L in two cases EL and EN are the results. Therefore E cannot be 5 because if R and L are 9 and 7, 7x5 equals 35 and even after adding a 9 a number in the fifties cannot be reached. Therefore E must be 1. In column II, lines 4 and 5, we see that E (or 1) plus N gives us 1. Therefore, N must be 0 or 9. But N cannot be 9 since 1 would have to be carried to column I, in which case E plus 1 (or 1 plus 1) would equal E. Therefore N must be 0.

			1	B	C	1
				S	R	L
			C	0	D	L
	1	L	L	G	R	
1	0	S	1	S		
1	1	V	S	L	G	L

The S in line 2 times the C in line 1, gives us a number ending in 1 (line 5). The only factorable number ending in 1 below 72 (the product of 9 and 8) is 21. Therefore S and C are 7 and 3. But which is which? Suppose S is 3. Then S times 1 (line 1, column IV) equals 3. But in line 5, columns I and II, we see that it equals 10 and because we couldn't have 7 to carry (3x9 would only leave 2 to carry) S cannot be 3. Therefore S equals 7 and C equals 3. So far, this is what we have:

			1	B	3	1
				7	R	L
			3	0	D	L
	1	L	L	G	R	
1	0	7	1	7		
1	1	V	7	L	G	L

Line 5 is complete. We can therefore find B, because 7x1B31 equals 10717, so that B must be 5. Adding in column IV, 3 plus L plus 1 equals 7, or 3 plus L plus 1 plus 1 (the latter from the previous column) equals 7. Solving the first equation L equals 3. But this cannot be since 3 is already C. Therefore the other equation is correct and L equals 2.

Multiplying 2 by 1531 we get 3062. Therefore D (line 3) equals 6. When we add in column VI we have to carry 1 because D is 6 and R must be above 3 (1, 2, and 3 are already taken). Therefore in column V, 0 plus G plus 7 plus 1 (previous column) equals 12. We find that G thus equals 4. In column III, 2 plus 7 equals V. Therefore V equals 9. Because 8 is the only unused digit, R equals 8 and the finished problem looks like this:

			1	5	3	1	
				7	8	2	
			3	0	6	2	
	1	2	2	4	8		
1	0	7	1	7			
1	1	9	7	2	4	2	which checks.

Now let us try our puzzle in division:

							D	D	G	
C	F	C	)	C	B	B	H	Y	G	
				E	B	D				
					E	Y	Y			
					E	B	D			
						Y	H	G		
						Y	H	G		

Although we don't have much to start with, we see that in the first subtraction we have:

	C	B	B	H			E	B	D
—		E	B	D	or		+	E	Y
			E	Y		C	B	B	H

From this we see that something must have been carried over to E in the hundreds column because CB is greater than E. Since 1 is the largest number that can be carried

over, E equals 9 and CB equals 10.

In the second subtraction we notice that Y minus zero equals Y and therefore nothing has been borrowed from Y in the minuend. Therefore Y minus D equals H.

Since 1 was borrowed for H in the first subtraction, H stands for H plus 10, and therefore Y plus D equals H plus 10. Subtracting the second equation from the first:

$$\begin{array}{r} Y-D=H \\ Y+D=H+10 \\ \hline -2D=-10 \\ D=5 \end{array}$$

Our example now looks like this:

$$\begin{array}{rrrr|rrrrrr} & & & & & & & 5 & 5 & G \\ \hline 1 & F & 1 &) & 1 & 0 & 0 & H & Y & G \\ & & & & 9 & 0 & 5 & & & \\ \hline & & & & & 9 & Y & Y & & \\ & & & & & 9 & 0 & 5 & & \\ \hline & & & & & & Y & H & G & \\ & & & & & & Y & H & G & \\ \hline \end{array}$$

In the second multiplication we see that 5 times $1F1$ equals 905. Consequently $1F1$ equals 181 or F equals 8.

Up to this point it has not been too difficult. Now the going begins to get tough. In the second subtraction we see that Y equals H plus 5. Therefore the only possibilities are:

H equals 1, 2, 3, or 4, because H must be less than 5 to get a 1 to carry to the next column so that 1 from zero (or 10) should equal 10.

Therefore: Y equals 6, 7, 8, or 9.

But 1, 8, and 9 are already known so that also eliminates their complements 6, 3, and 4. From this we see that the only possible solution is H equals 2 and Y equals 7.

Now the only letter left is G. In the last multiplication we see that: 181 x G=72G.

Since G times one cannot be greater than 9, nothing is carried over, and 18 times G equals 72. G therefore equals 4.

Finally we get the completed example, which looks like this:

$$\begin{array}{rrrr|rrrrrr} & & & & & & & 5 & 5 & 4 \\ \hline 1 & 8 & 1 &) & 1 & 0 & 0 & 2 & 7 & 4 \\ & & & & 9 & 0 & 5 & & & \\ \hline & & & & & 9 & 7 & 7 & & \\ & & & & & 9 & 0 & 5 & & \\ \hline & & & & & & 7 & 2 & 4 & \\ & & & & & & 7 & 2 & 4 & \\ \hline \end{array}$$

This also checks.

Do you think you can solve these problems *now?* Try the examples on Page 13 and see how good you *really* are.

MATH ACTIVITIES

In addition to the regular curriculum, the Mathematics Department sponsors a number of extra-curricular activities. These five, the Math Club, the Math Help Squad, the Math Team, the Math Kraft Club, and the Slide Rule Club are designed for the Technite, whatever the degree of his mathematical knowledge.

Math Club: For those Technites interested in mathematics "higher" than that normally taught in the classroom, there is the Math Club. The club meets each Tuesday afternoon in room 5S5. Mr. D. Malament is the faculty adviser.

Math Help Squad: The Math Help Squad was organized for the purpose of assisting students deficient in mathematics. Since its organization, it has expanded to publish a mimeographed paper called "Math Notes" and to organize a lunchroom squad, the *Matheteria*. The squad meets Tuesday, Wednesday, and Thursday afternoons in room 5W24 for Math I and II and in room 5E2 for the remaining six terms of math. Mr. F. Zuckerman is the faculty adviser.

Math Team: Five times a term the Brooklyn Tech Math Team meets to compete in the Interscholastic Mathematics League contests. Preparation is obtained through Mr. S. Kleinberg's *Math Analysis* class. Admission to the class, which meets daily during the eighth period, is determined by Mr. Kleinberg on the basis of the applicant's previous mathematics record.

Math Kraft Club: The Math Kraft Club is organized for the student interested in constructing projects of mathematical interest. The faculty adviser is Mr. S. Sklar.

Slide Rule Club: Practice in the use of the slide rule as well as various tricks in its use is afforded by the Slide Rule Club. The club meets on alternate Wednesdays in room 6E2. Mr. E. Davis is the faculty adviser.

PROBLEM PAGE

Solutions

(*Problems on Page* 13)

Literal division and multiplication problems: 1071797÷109=9833 and 501×876=438876.